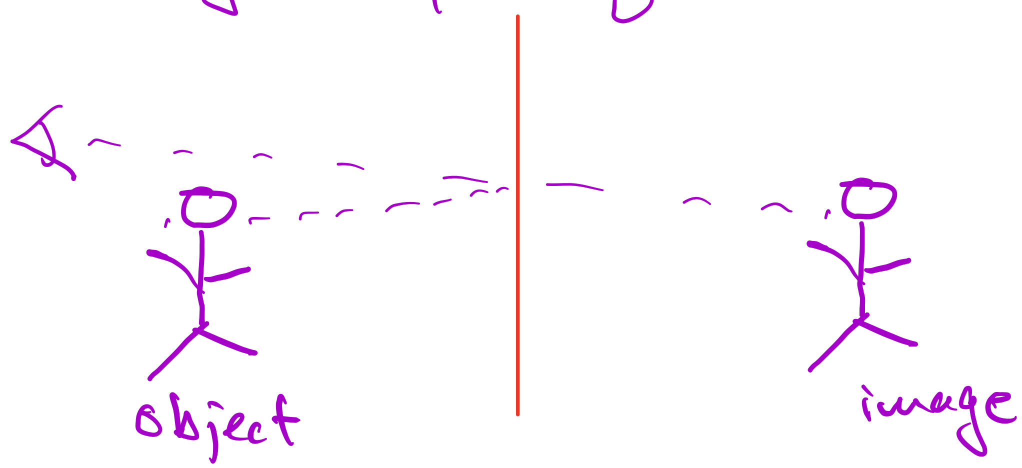
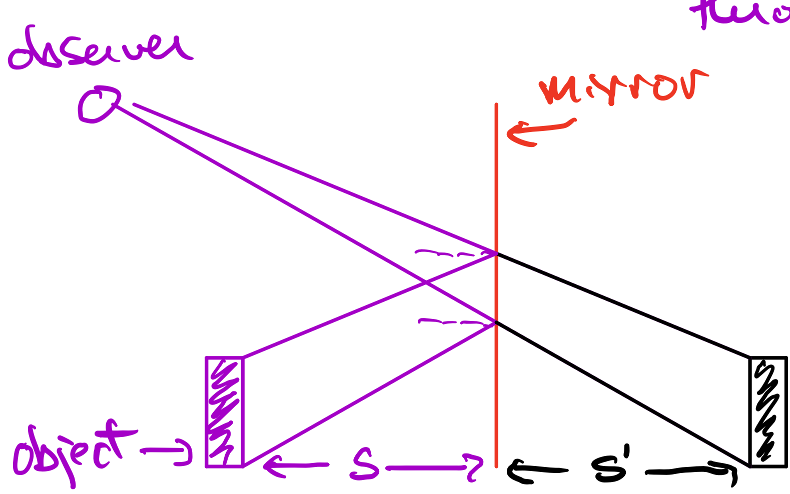
Using reflection & refraction

we extrapolate along straight lines

place figure in front of mirror



the image is "virtual" – no light passes through it

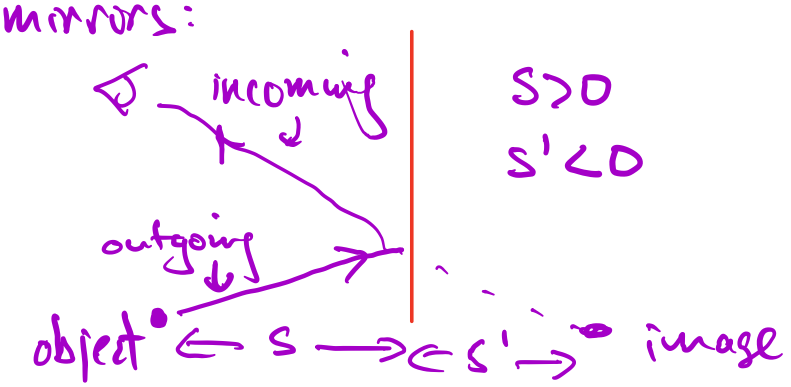


1. use s = distance of object from mirror
2. put observer anywhere you like
3. draw rays from object to observer using law of reflection: $\theta_r = \theta_i$
4. find image distance s' by extending rays

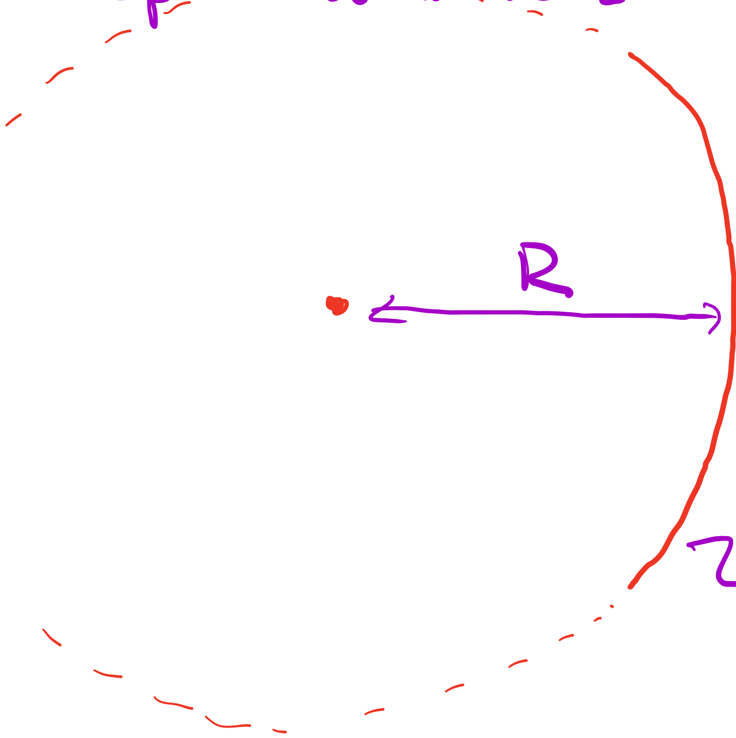
convention for distances:

1. $s > 0$ means object is on same side of surface as incoming light
2. $s' > 0$ means image is on same side of surface as outgoing light
3. for curved surfaces, radius of curvature $r > 0$ if center of curvature is on same side as outgoing light

for mirrors:



Spherical mirrors

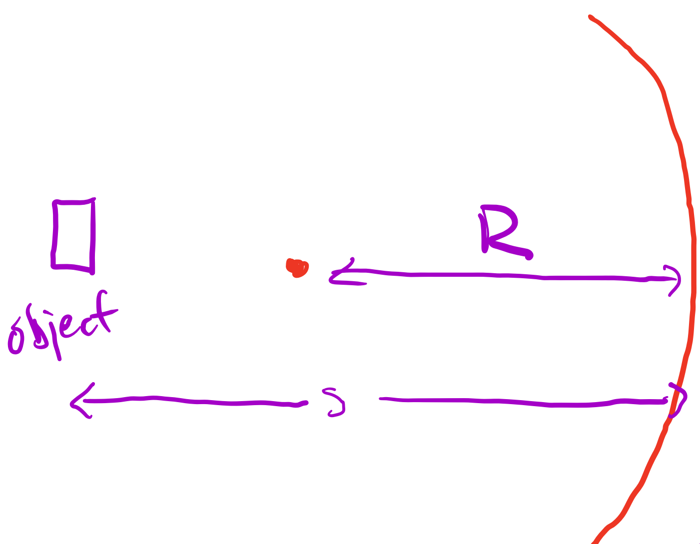


R = radius of curvature

if object is on same side as center, $R > 0$

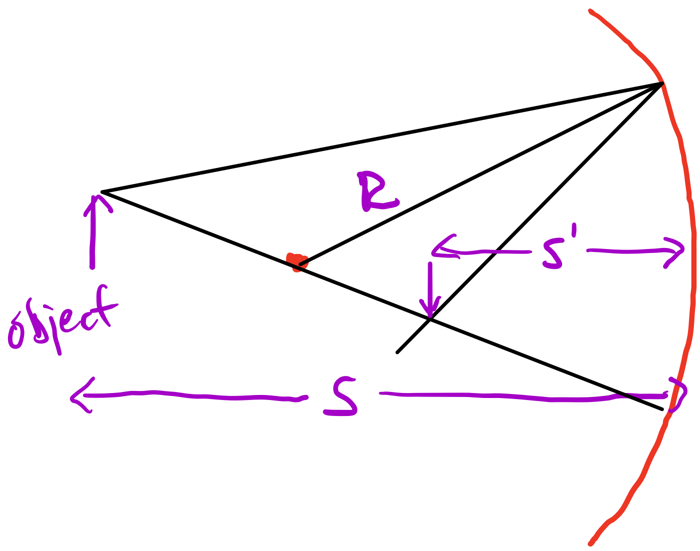
← concave: opens to the object

flip it around & its convex

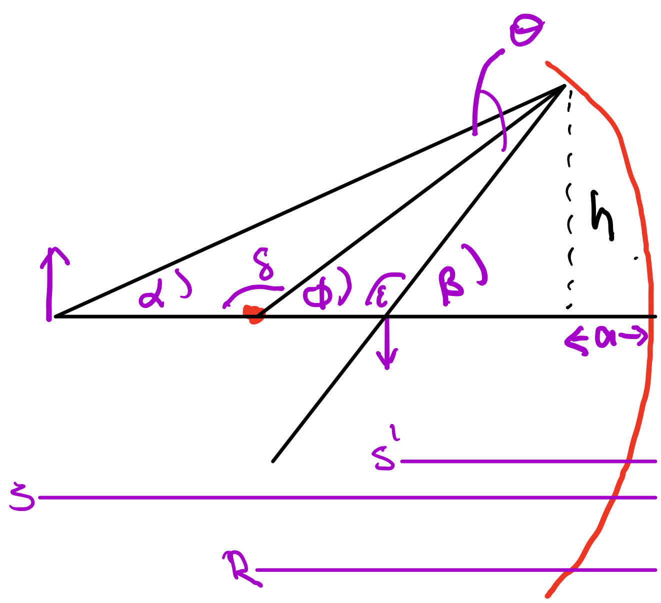


light from any point on object will reflect around normal to surface

$\Rightarrow$ normal goes thru center



To calculate relationship between s, s', R



$\alpha + \theta + \delta = \pi$ and $\phi + \delta = \pi$ $\therefore$ $\alpha + \theta = \phi$

$\phi + \theta + \varepsilon = \pi$ and $\beta + \varepsilon = \pi$ $\therefore$ $\phi + \theta = \beta$

subtract: $\alpha - \phi = \phi - \beta \Rightarrow \alpha + \beta = 2\phi$

then: $\tan\beta = \frac{h}{s'-a}$ $\tan\phi = \frac{h}{R-a}$ $\tan\alpha = \frac{h}{s-a}$

if we restrict rays so that α is small, then $a \to 0$ and α, β, ϕ are all small

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\lim_{\theta \to 0} \sin\theta \sim \theta$

$\lim_{\theta \to 0} \cos\theta \sim 1 - \theta^2/2 \sim 1$

so $\lim_{\theta \to} \tan\theta = \theta$

this gives $\tan\beta \sim \beta = \frac{h}{s'}$

$\tan\phi \sim \phi = \frac{h}{R}$

$\tan a \sim a = \frac{h}{s}$

next plug into $\alpha + \beta = 2\phi$

$$\frac{h}{s} + \frac{h}{s'} = \frac{2h}{R}$$

gives $\boxed{\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}}$

this eqn is only approximate for small angles

→ so make mirrors smaller and all angles will be small. or make them parabolic!

deviations from above equation results in spherical abberations → image blurs

can be corrected by changing shape of mirror to make above eqn exact

Rule for geometric optics:

1. draw ray from any pt on object, parallel to symetry axis
2. reflect using $\theta_r = \theta_i$
3. draw ray from same pt thru center of curvature - will reflect back

onto itself

4. intersection is where that point sits on image

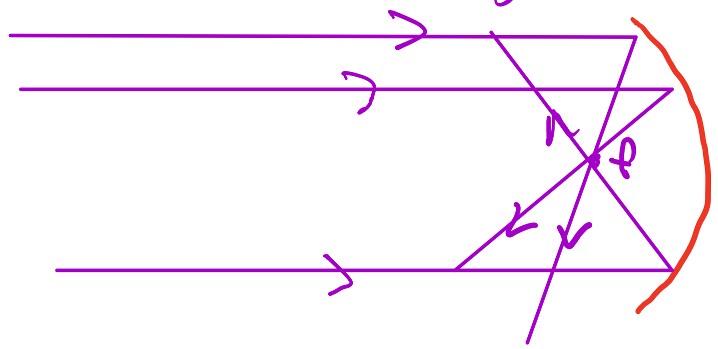
Focal point:

if $s \to \infty$, then $\frac{1}{s} \to 0$ so $\frac{1}{s'} = \frac{2}{R}$

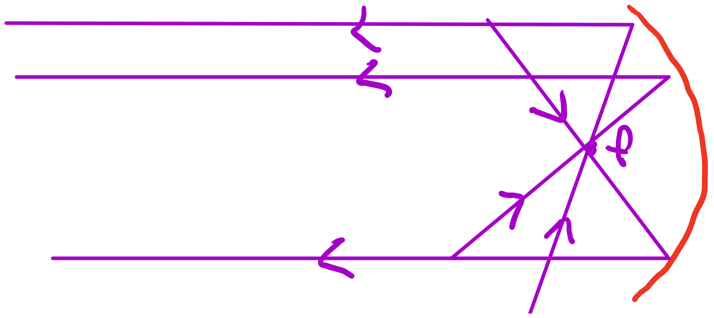
$s' = R/2$

this is called "focal point" → all objects at $s = \infty$ produce image at $s' = f$ where $f = R/2$

for spherical mirror, f = focal length



and it works in reverse: any incoming ray thru f will reflect parallel to axis, out



this allows you to find images easily

Magnification

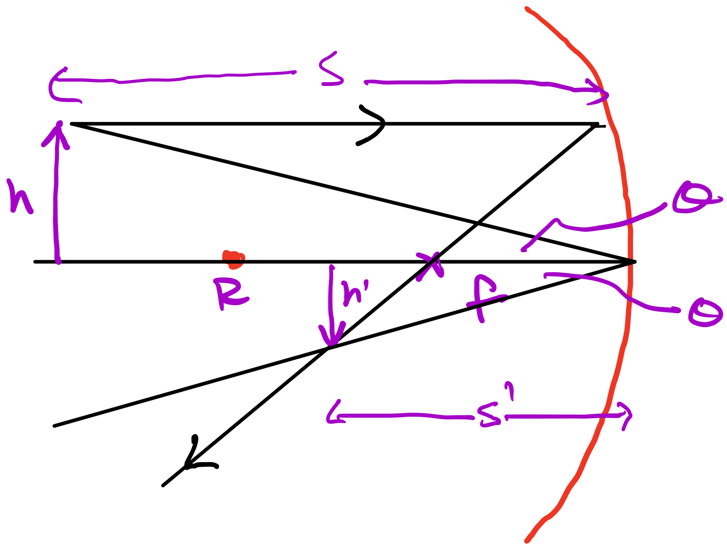


image is

1. inverted
2. real $(s'>0)$
3. magnification:

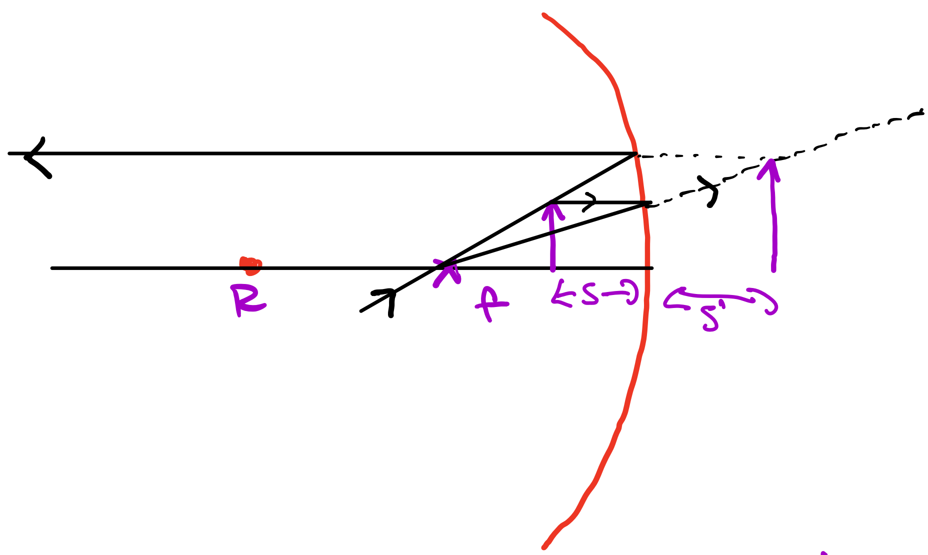
from upper triangle: $\tan\theta = h/s$
from lower " $\tan\theta = h'/s'$

magnification is $\boxed{m = \frac{h'}{h} = \frac{s'}{s}}$

since h' is <0 (below axis) write

$$m = \frac{h'}{h} = -\frac{s'}{s}$$

Concave mirror w/ object inside f



1. draw ray coming in thru f → goes out parallel
2. draw ray parallel to mirror → reflects as if it goes out thru f

$$\frac{1}{s'} + \frac{1}{s} = \frac{1}{f} \Rightarrow \frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{s-f}{sf}$$

$$s' = \frac{1}{\frac{1}{f} - \frac{1}{s}} = \frac{fs}{s-f}$$

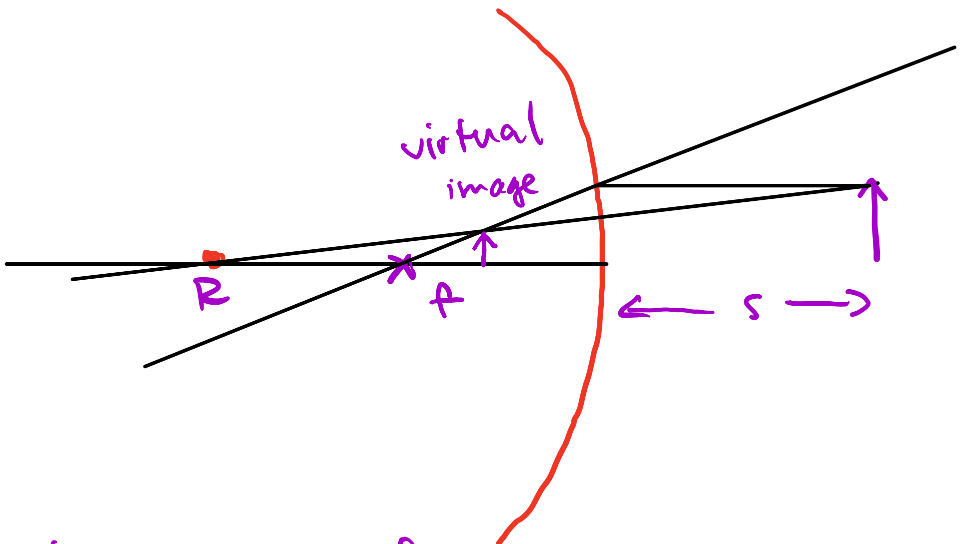
since $s<f$, $s'<0$ so is on other side of mirror and image is virtual

(no light actually goes thru it)

$$m = -\frac{s'}{s} = \frac{f}{s-f}$$

as you move object closer to focal pt, image gets bigger

<u>convex mirrors</u>



here $R<0$ so $f<0$

$$\frac{1}{s'} = \frac{-1}{f} - \frac{1}{s} = -\frac{(s+f)}{sf}$$

$$s' = -\frac{sf}{s+f} < 0$$ virtual image, behind mirror
no light goes thru image

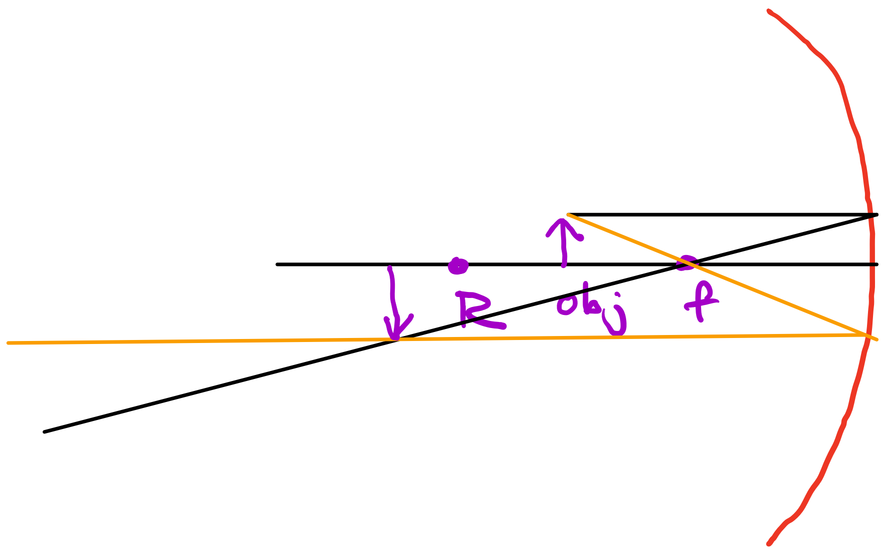
magnification

$$m = -\frac{s'}{s} = \frac{f}{s+f} < 1$$ image smaller than object

example: concave mirror, $R = 10\,cm$, $s = 8\,cm$

1. image of the base will be on the horizontal
2. " of head using these 2 rules. start at head of arrow and do this:

⇒ draw line parallel to mirror ⇒ will reflect thru focal pt $f = R/2 = 5\,cm$



⇒ draw line thru focal pt will diverge parallel (to ∞)

intersection will be where image forms

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{5} - \frac{1}{8} = \frac{3}{40} \Rightarrow s = \frac{40}{3}\,cm = 13.3\,cm$$

$$m = -\frac{s'}{s} = -\frac{13.3}{8} = -1.66$$

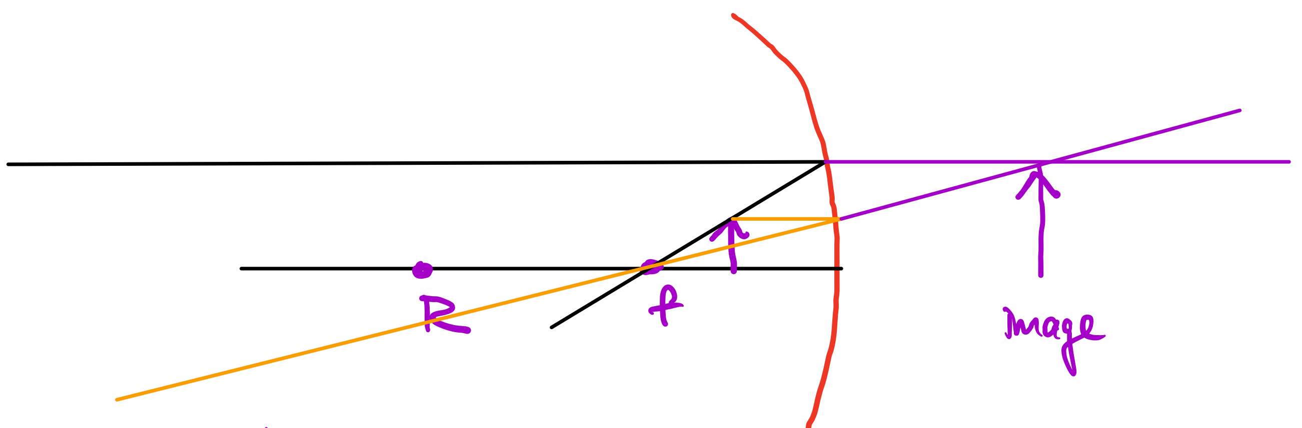
$s' > 0 \therefore$ image is real

$m < 0 \therefore$ image is inverted

note: $\frac{1}{s'} = \frac{1}{f} - \frac{1}{s}$ if $f < s$, $\frac{1}{s} > \frac{1}{f} \therefore s' > 0$ real

if $s < f$ (object inside focal pt)

image will be virtual

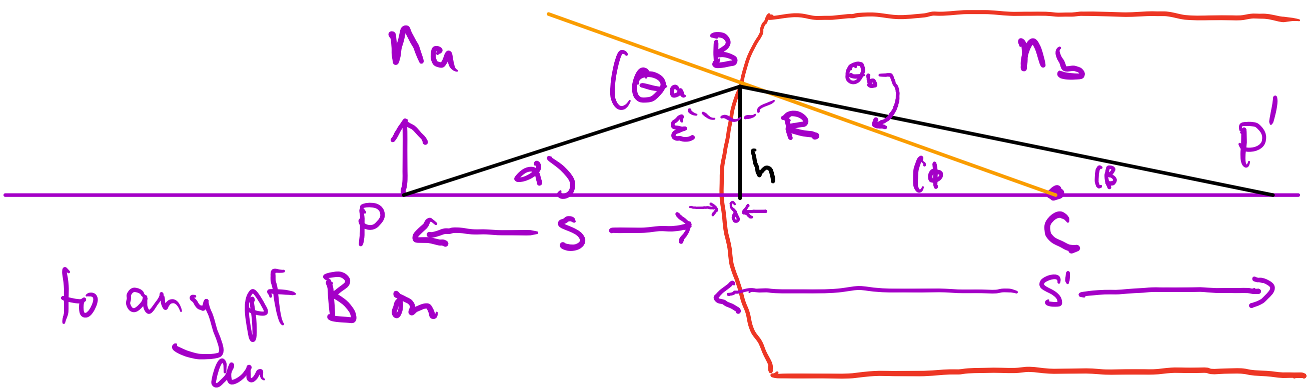


see how 2 rays diverge on real side!

We've seen how mirrors + geometry + law of reflection allows us to have images w/ various characteristics.

$\Rightarrow$ can we do the same or something similar w/ refraction?

Refraction at spherical surface



to any pt B on
an

- ray from base point P to a point B on surface
  $\Rightarrow$ makes angle θ_a wrt normal
  $\Rightarrow$ makes angle α wrt horizontal
- ray refracts, angle θ_b, and intersects horizontal at point P'
  $\Rightarrow$ makes angle β wrt horizontal
- normal goes thru center of curvature C and has length R, angle ϕ wrt horizontal
- distance of B to horizontal is h
  $\Rightarrow$ δ is distance from base of h to front of lens

index of refraction n_a on object side, n_b on outgoing side

$\Rightarrow$ here $n_b > n_a$

same sign convention as before: $R > 0$ if curvature is on outgoing side

just like before: $\phi = \theta_b + \beta$

$\theta_a = 180^\circ - \varepsilon$ and $\alpha + \phi + \varepsilon = 180$

so $\theta_a = \alpha + \phi$

also from law of refraction:

$$n_a \sin\theta_a = n_b \sin\theta_b$$

for "small angles", $\sin\theta \sim \theta$

so $n_a\theta_a = n_b\theta_b$

also $\tan\alpha = \dfrac{h}{s+\delta}$ $\tan\beta = \dfrac{h}{s'-\delta}$ $\tan\phi = \dfrac{h}{R-\delta}$

if angles are small then $\delta \rightarrow 0$

and $\tan\theta \sim \theta$

this gives:

$n_a\theta_a = n_b\theta_b$

$$\alpha = \frac{h}{s} \qquad \beta = \frac{h}{s'} \qquad \phi = \frac{h}{R}$$

$$\phi = \theta_b + \beta \Rightarrow \frac{h}{R} = \theta_b + \frac{h}{s'} \Rightarrow \theta_b = \frac{h}{R} - \frac{h}{s'}$$

$$\theta_a = \alpha + \phi \Rightarrow \theta_a = \frac{h}{s} + \frac{h}{R}$$

$$n_a\theta_a = h\,n_a\left(\frac{1}{s} + \frac{1}{R}\right) = n_b\theta_b = h\,n_b\left(\frac{1}{R} - \frac{1}{s'}\right)$$

cancel out h:

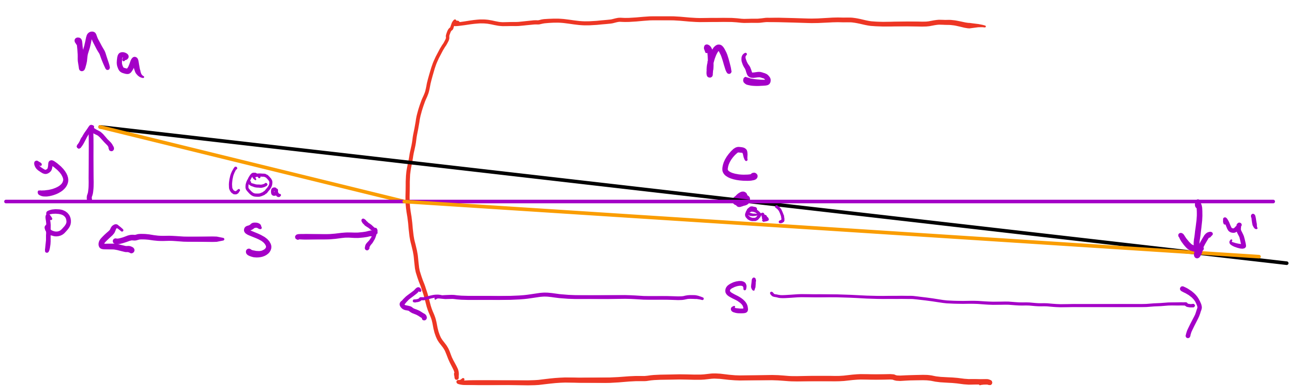
$$\frac{n_a}{s} + \frac{n_a}{R} = \frac{n_b}{R} - \frac{n_b}{s'}$$

or

$$\boxed{\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}}$$

This formula is only good for "small angles"

Now draw extended object - rays from arrow part



- ray thru center of curvature C goes straight
- ray thru point V is refracted towards normal, which is the horizontal

$\tan\theta_a = y/s$ and $\tan\theta_b = y'/s'$

for small angles $\tan\theta \approx \sin\theta \approx \theta$

so $\theta_a s = y$ and $\theta_b s' = y'$

$$m = \frac{-y'}{y} = -\frac{s'\theta_b}{s\theta_a}$$

law of refraction for small angles:

$$n_a\theta_a = n_b\theta_b \quad \text{or} \quad \frac{\theta_b}{\theta_a} = \frac{n_a}{n_b}$$

so $m = -\frac{s'}{s}\frac{\theta_b}{\theta_a} = -\frac{s'}{s}\frac{n_a}{n_b}$

since s' is in region b we can write

$$m = -\frac{(s'/n_b)}{(s/n_a)}$$

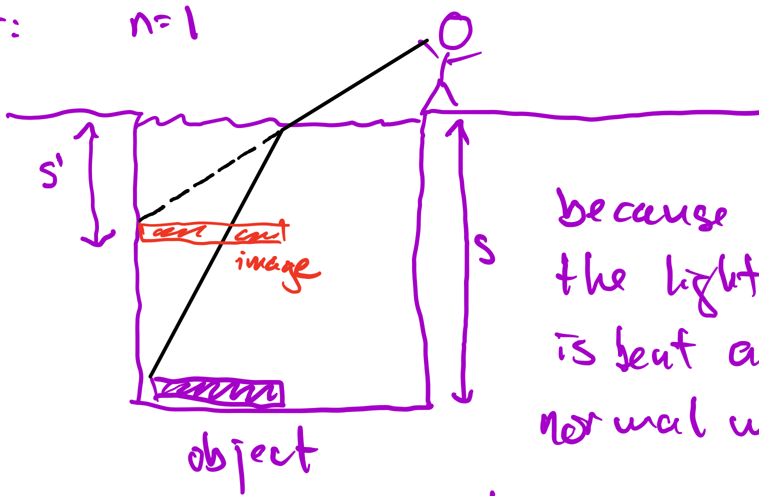
same convention: $s' > 0$ real image
$s' < 0$ virtual
$m > 0$ inverted image

for flat surfaces, $R = \infty$

$$\frac{n_a}{s} + \frac{n_b}{s'} = 0$$

this allows you to image for flat surface

ex: $n = 1$



because of refraction, the light from object is bent away from normal when you see it

if pool has depth s, w/object at bottom, image is formed at depth s'.

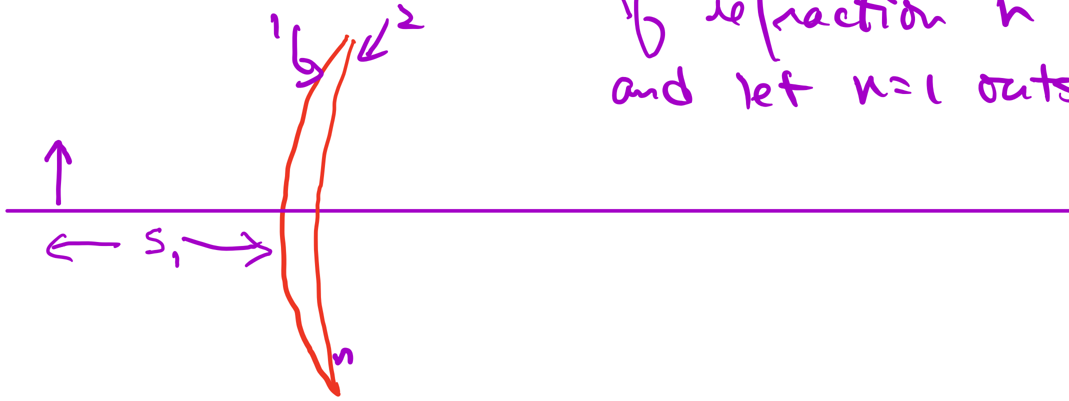
$\Rightarrow$ if image $s' < 0$ then image is in water but to use the equations assume image is in air

$$\frac{1}{s'} + \frac{1.33}{s} = 0 \Rightarrow s' = -\frac{s}{1.33} < 0$$ image is virtual & in water

if pool is 2m deep, $s' = \frac{2}{1.33} = 1.5m$

$\Rightarrow$ pool is not as deep as it looks!

Imaging thin materials → 2 surfaces w/ index of refraction n inside and let n=1 outside



equation for image due to 1st surface:

$$\frac{1}{s_1} + \frac{n}{s_1'} = \frac{n-1}{R_1}$$

the image formed by R_1 surface becomes the object for R_2

note: that image will <u>not</u> be on incoming side of light!

$$\frac{n}{s_2} + \frac{1}{s_2'} = \frac{1-n}{R_2}$$

s_2 is the object formed by image of surface R_1 and $s_2 = -s_1'$ because that object is <u>not</u> on the same side as incoming light

2 equations:

$$\frac{1}{s_1} + \frac{n}{s_1'} = \frac{n-1}{R_1}$$

$$\frac{n}{s_2} + \frac{1}{s_2'} = -\frac{n}{s_1'} + \frac{1}{s_2'} = \frac{1-n}{R_2}$$

we want to know s_2' so eliminate s_1' by adding the equations

$$\frac{1}{s_1} + \frac{1}{s_2'} = \frac{n-1}{R_1} + \frac{1-n}{R_2} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

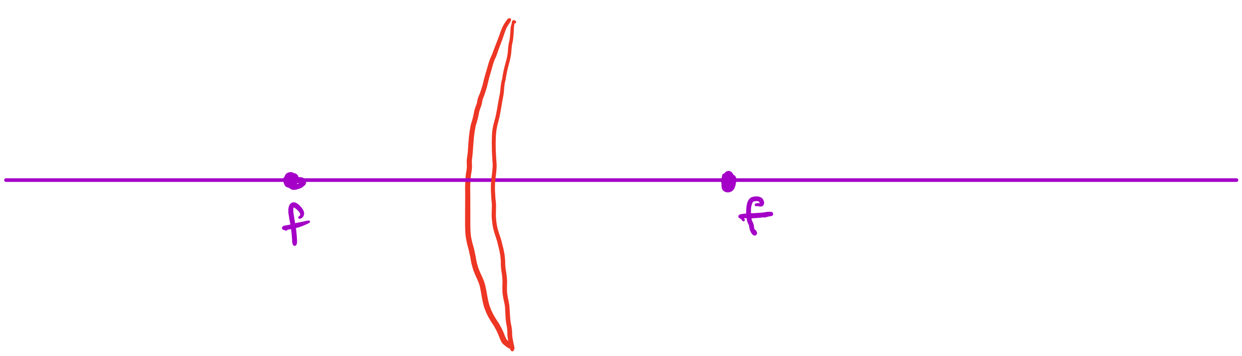
focal pt is as before: point on optical axis that rays from ∞ converge to

at $s = \infty$, $s' = f$ so $\boxed{\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$

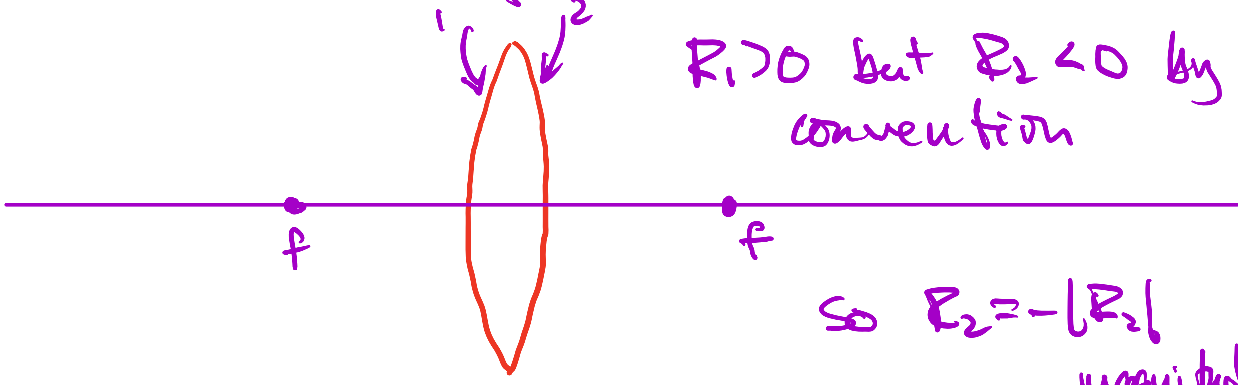
this is "lens maker's eqn"

thin lens equation: $\boxed{\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}}$

note: focal pt is symmetric for rays hitting from left <u>or</u> right because it only depends on n and the shape of the 2 surfaces



A more interesting lens:



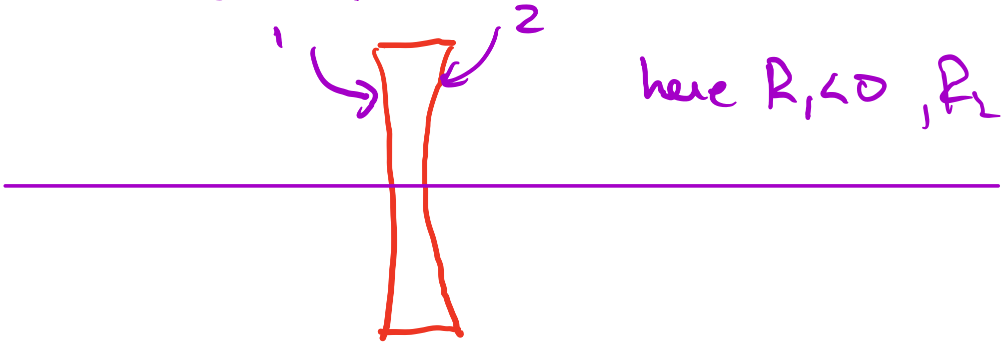
$R_1 > 0$ but $R_2 < 0$ by convention

so $R_2 = -|R_2|$ magnitude

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{-|R_2|}\right)$$

$$= (n-1)\left(\frac{1}{R_1} + \frac{1}{|R_2|}\right)$$

what about this:



here $R_1 < 0$, $R_2 > 0$

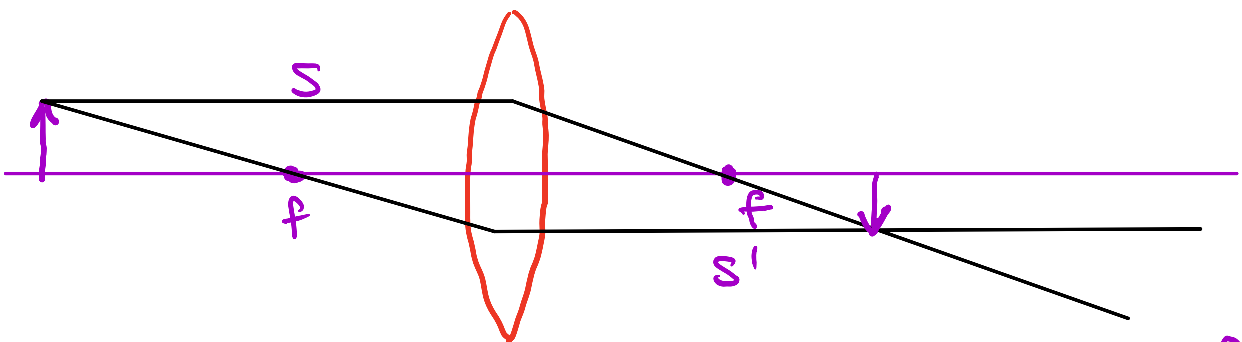
so $$\frac{1}{f} = (n-1)\left(\frac{-1}{|R_1|} - \frac{1}{R_2}\right) = -(n-1)\left(\frac{1}{|R_1|} + \frac{1}{R_2}\right)$$

$f < 0$ for "diverging" lens

Ray tracing:

1. parallel light is bent thru f
2. light thru f is bent parallel
3. light thru cent of lens along optical axis is unbent

ex: real image outside f



$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{s-f}{fs} \Rightarrow s' = \frac{fs}{s-f} > 0 \text{ if } s > f$$

image is real - light goes thru it!

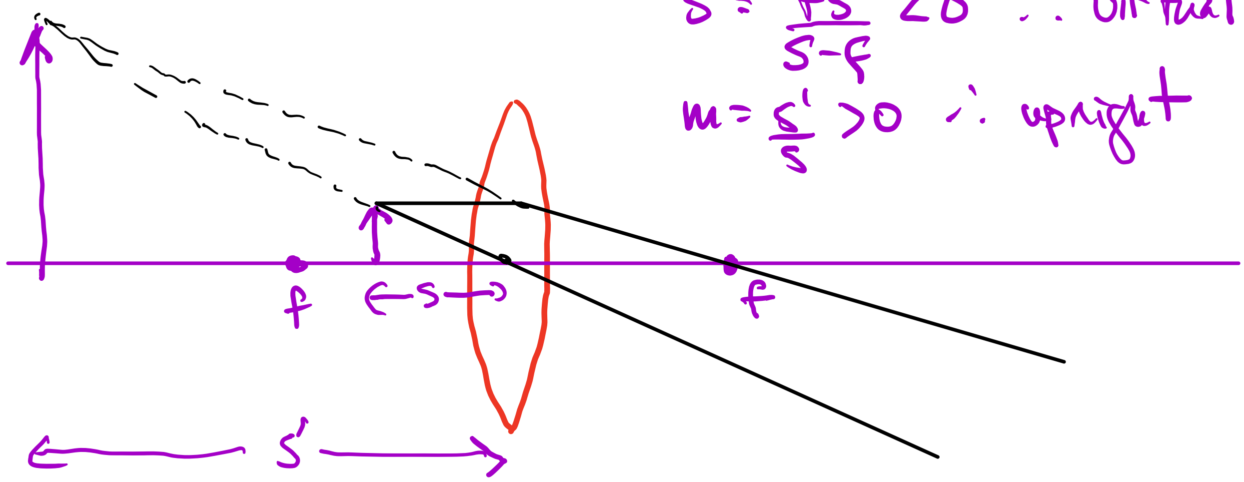
$$m = -\frac{s'}{s} = -\frac{f}{s-f}$$ image is inverted

This is a converging lens!

$s < f$ (inside f)

$$s' = \frac{fs}{s-f} < 0 \quad \therefore \text{ virtual}$$

$$m = \frac{s'}{s} > 0 \quad \therefore \text{ upright}$$



you can see that as s gets close to f from inside, virtual image is magnified more

Diverging lens: negative focal pt.

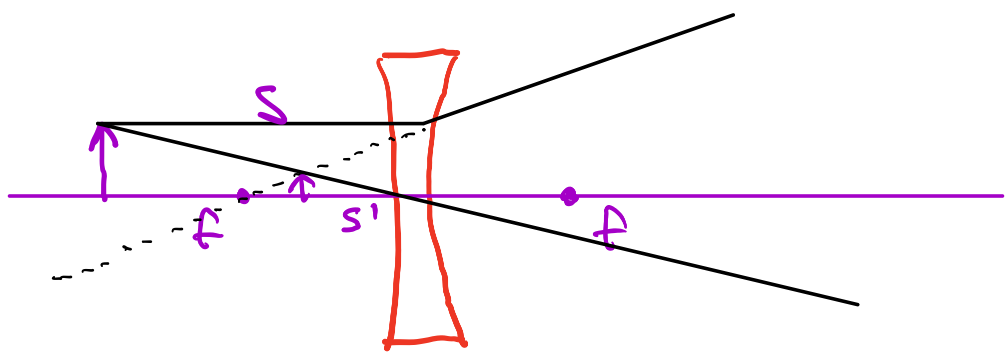
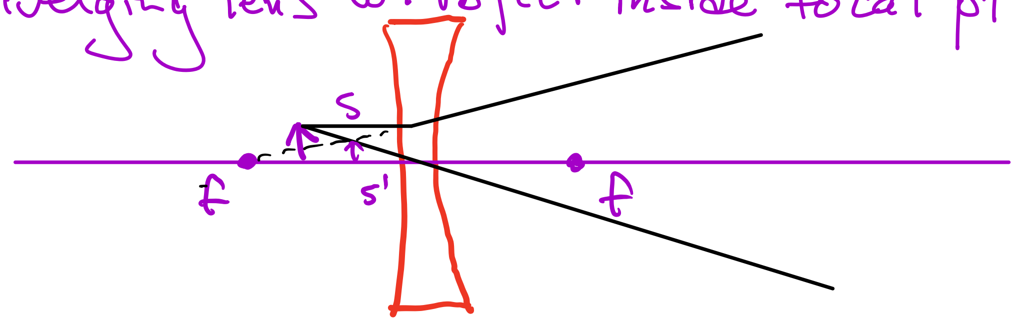


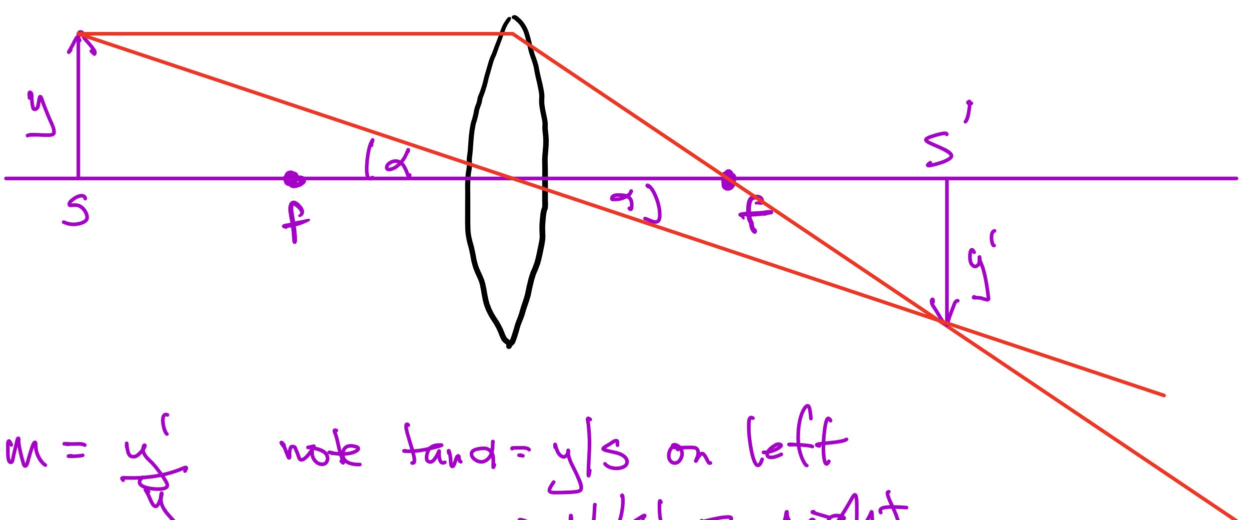
image is virtual, $s' < 0$, upright

Diverging lens w/ object inside focal pt



no large difference for either side of f

Magnification



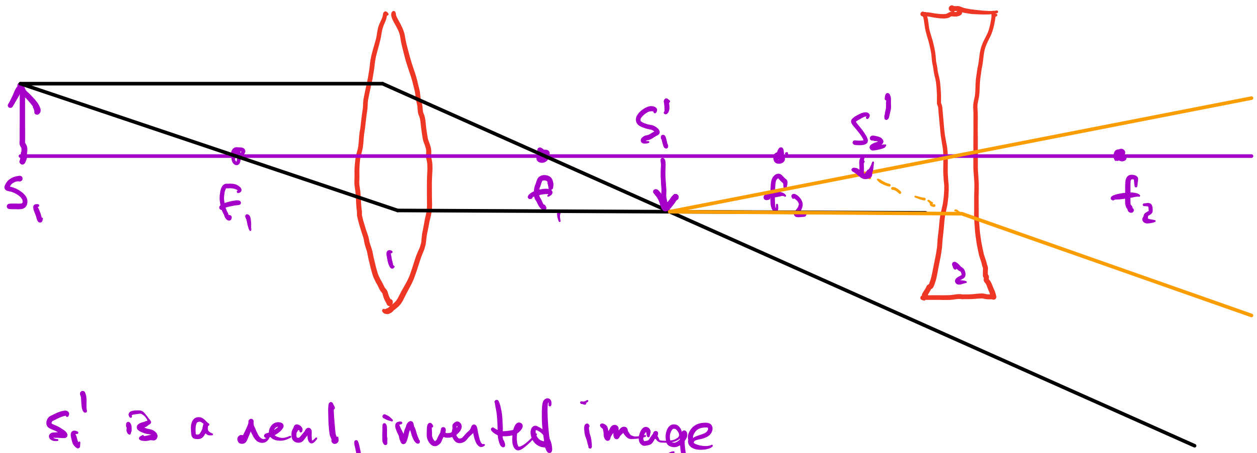
$m = \frac{y'}{y}$ note $\tan\alpha = y/s$ on left

$= y'/s'$ on right

so $\frac{y}{s} = \frac{y'}{s'}$ so $\frac{y'}{y} = \frac{s'}{s}$

$m = -s'/s$ minus sign to denote above or below horizontal axis

Compound lenses



s_1' is a real, inverted image

$\Rightarrow$ light goes thru lens 2

let $d = 1m$ separate lenses

$f_1 = 25cm$ & $f_2 = -25cm$

$s_1 = 50cm$ from lens 1

1st image: $\frac{1}{s_1'} = \frac{1}{f_1} - \frac{1}{s} = \frac{s - f_1}{sf_1} = \frac{50-25}{50 \cdot 25} = \frac{25}{50 \cdot 25} = \frac{1}{50}$

$s_1' = 50cm > 0 \therefore$ to right of lens 1

2nd lens: $s_2 = d - 50$ to left of lens 2

$= 1 - 50 = 50cm$ to left

$\frac{1}{s_2'} = \frac{1}{f_2} - \frac{1}{s_2} = -\frac{1}{25} - \frac{1}{50} = -\frac{3}{50}$

$s_2' = -\frac{50}{3} = -16.7\,cm < 0 \therefore$

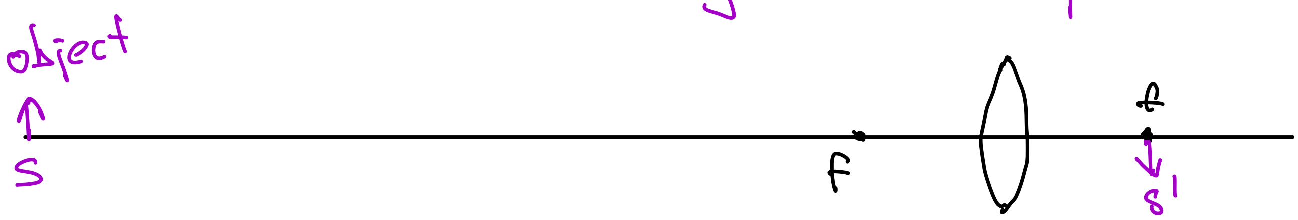
virtual, 16.7cm to left of lens 2

magnification $m = -\frac{s'}{s} = \frac{50/3}{50} = \frac{1}{3}$ smaller!

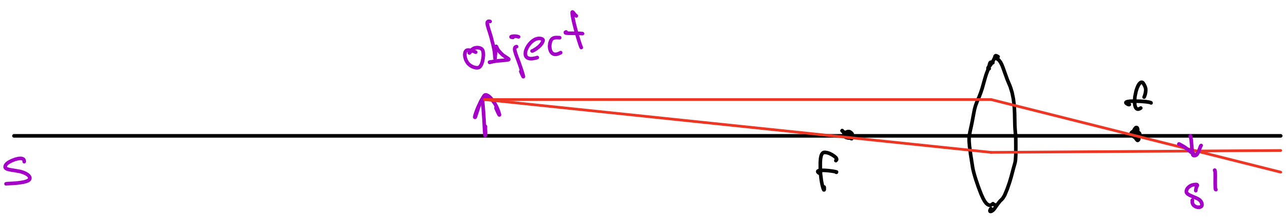
Lenses, basic usage

put object at ∞: $\frac{1}{s} = 0 \therefore \frac{1}{s'} = \frac{1}{f}$ or $s' = f$

image is at focal pt



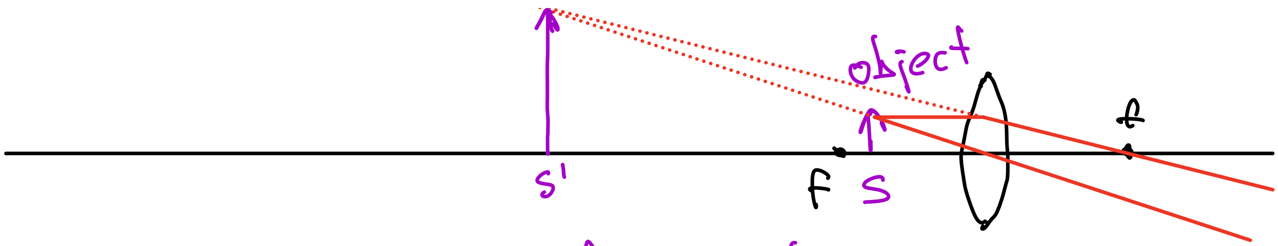
as you move object towards lens, image recedes away from f



bring object to focal pt, image is at ∞

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{f} - \frac{1}{f} = 0 \Rightarrow s' = \infty$$

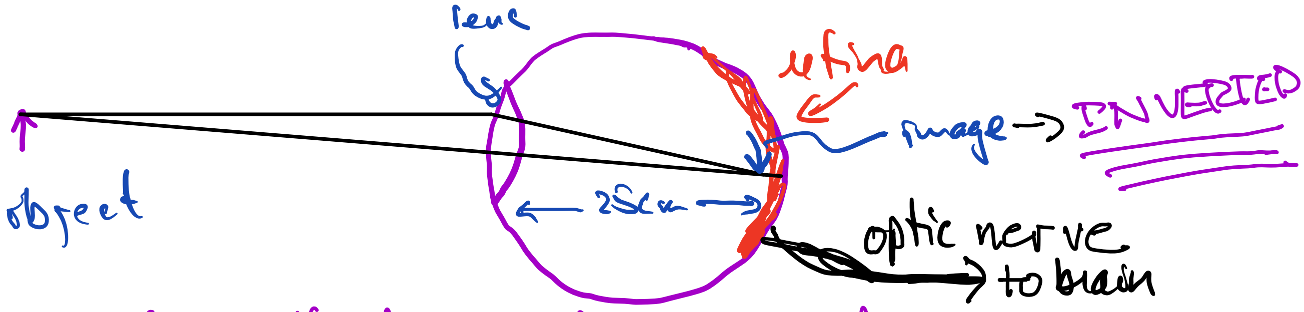
bring object <u>inside</u> focal pt, image swings around to object side & becomes virtual



this is how a magnifier works

Magnifier

size of an object determined by size of the image on retina



muscles attached to lens modifies the shape to focus objects on the retina which is behind the lens, about 2 cm

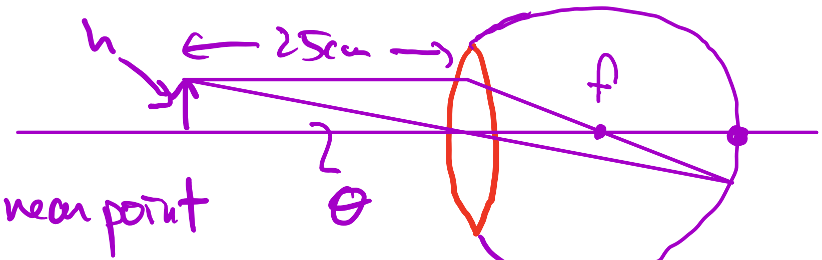
$\Rightarrow$ this is called "accommodation"

near point: closest you can bring object to eye and still accommodate $\rightarrow$ focus image on the retina

$\Rightarrow$ at near point, max accommodation

$\Rightarrow$ max reshaping lens by eye muscles

for most people this is ~25cm



at near point

eye adjusts shape, and f pt, so that image forms on the retina

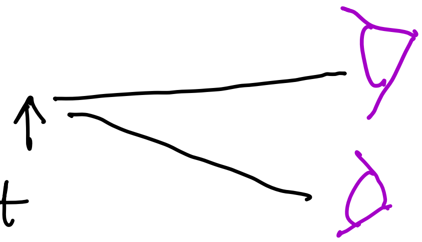
object subtends angle θ

$$\tan\theta \sim \theta = \frac{h}{25\text{cm}}$$

<u>and</u> at near point eyes are not seeing parallel

object at $\sim\infty$, eyes are ~parallel

object at near point, eyes have to look at angle towards object

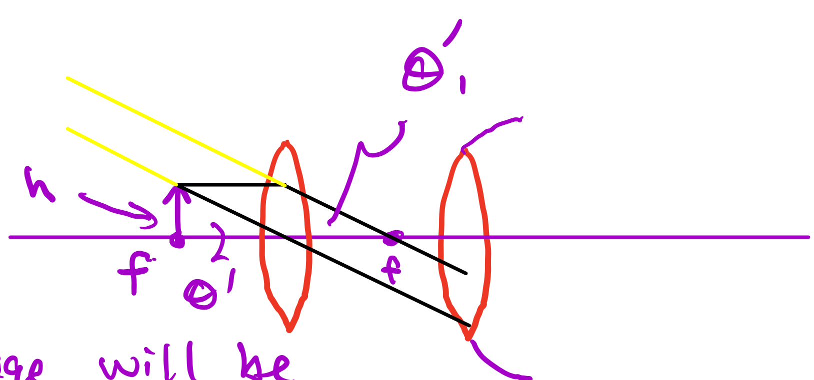


⇒ this can cause headaches!

<u>Glasses</u> act as magnifiers for close objects
as you age, near point moves outward
⇒ would be nice to read and not strain eyes to cross and accommodate too much

note: relaxed eye focal length $f \sim 2cm \sim$ distance between lens & retina

to accomplish:
⇒ place converging lens between object and eye such that object is at focal pt of that lens



the image will be virtual, at ∞, and eye can relax and focus onto retina.

Image will subtend angle θ'

$\tan\theta' = \theta' = h/f$

overall angular magnification:

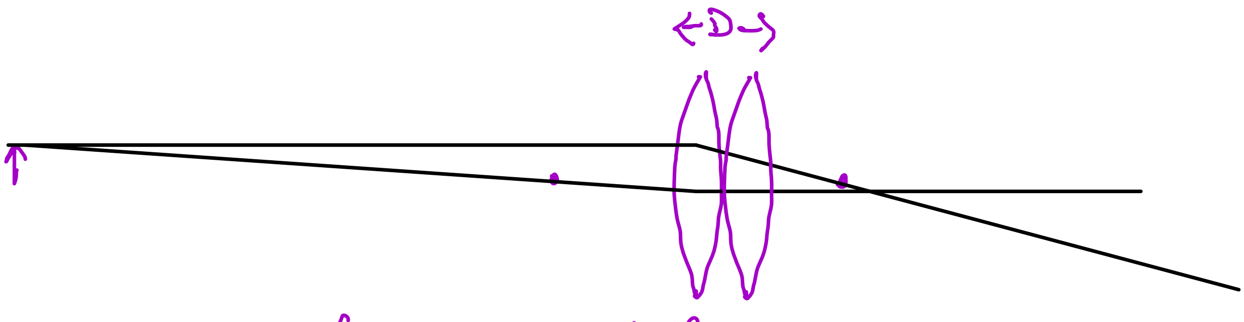
$$M = \frac{\theta'}{\theta} = \frac{h/f}{h/25cm} = \frac{25cm}{f}$$

the smaller the focal length of magnifier, the bigger the magnification; the closer you can bring the object

Diopter: $D = \frac{1}{f}$ for glasses

eg $D = 1.5 \Rightarrow$ focal length of $\frac{2}{3}m \sim 2$ ft

$D = 2.0 \Rightarrow f = \frac{1}{2}m \sim 1.5$ ft



lens 1: image for $s_1 = \infty$ is at f_1

lens 2: dist D behind 1, w/ f_2

$s_2 = -(f_1 - D) = D - f_1$ ($s_2 < 0$ since it is virtual)

Find image for lens 2: (with respect to lens 2)

$$\frac{1}{D-f_1} + \frac{1}{S_2'} = \frac{1}{f_2}$$

$$\frac{1}{S_2'} = \frac{1}{f_2} + \frac{1}{f_1 - D}$$

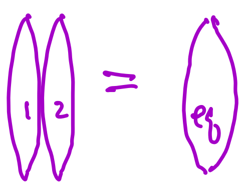
as $D \to 0$, $\frac{1}{S_2'} = \frac{1}{f_2} + \frac{1}{f_1}$

as if its 1 lens w/ focal pt $\frac{1}{f_{eq}} = \frac{1}{f_2} + \frac{1}{f_1}$

or $D_{eq} = D_1 + D_2$ for lenses, add diopters if they are "on top" of each other

ex: lens 1 has $f_1 = 50\,cm$, $f_2 = 75\,cm$

$D_1 = \frac{1}{50\,cm} = 2$ $\quad D_2 = \frac{1}{75\,cm} = 1.33$



$D_{eq} = 2 + 1.33 = 3.33$

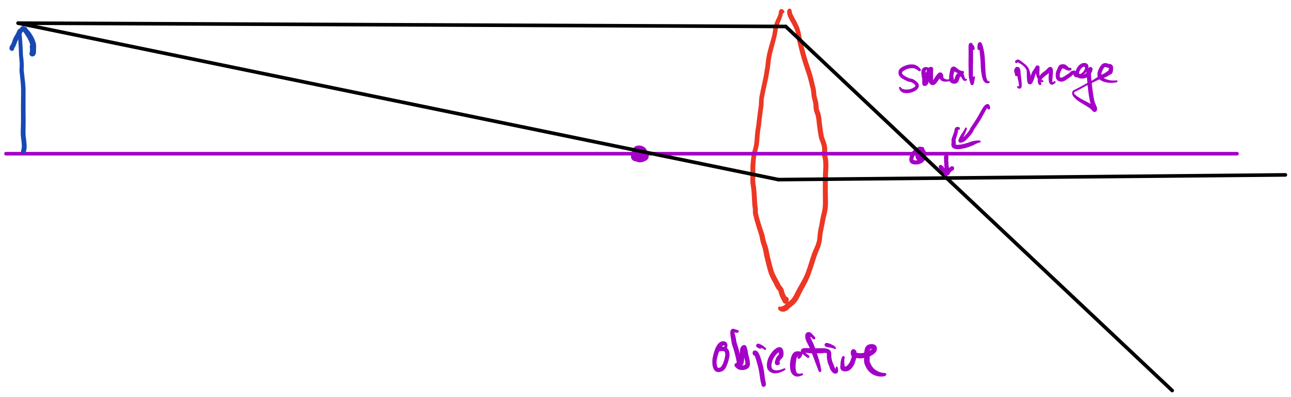
$f_{eq} = \frac{1}{3.33} = 0.3\,m$

so if you need a magnifier to see fine print, borrow another pair of glasses!

Compound lenses → 2 lenses

place object far away from 1^{st} lens called "objective"

image will be real, inverted, and "small"

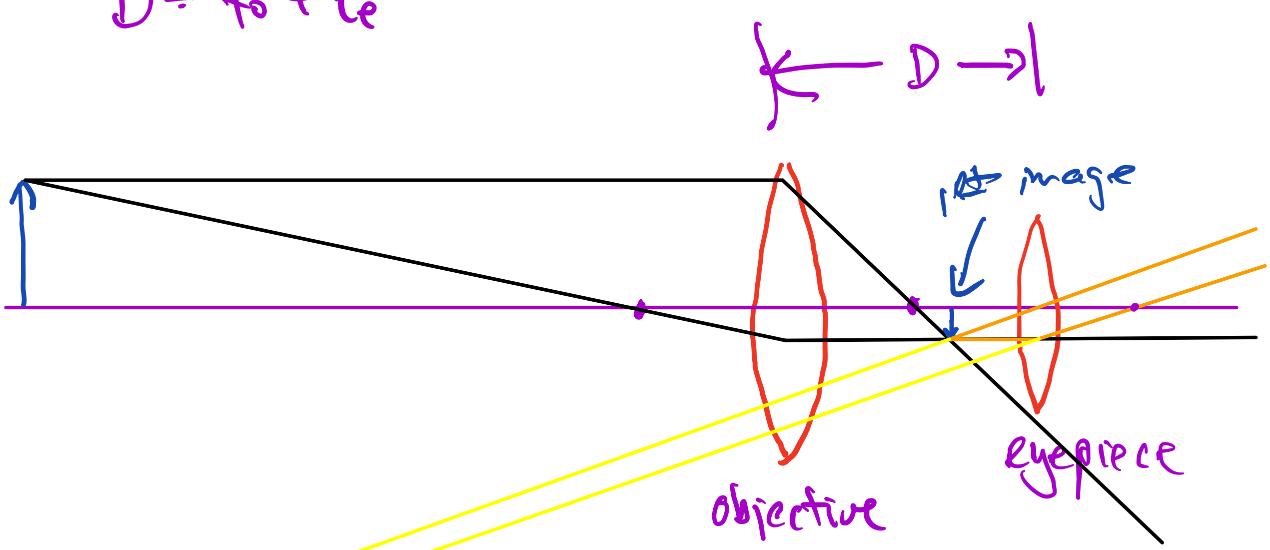


now add "eye piece" converging lens to use objective lens image as object

⇒ this is the "eyepiece" lens

construct so that objective image is inside (or at) eyepiece focal point

$D \simeq f_o + f_e$

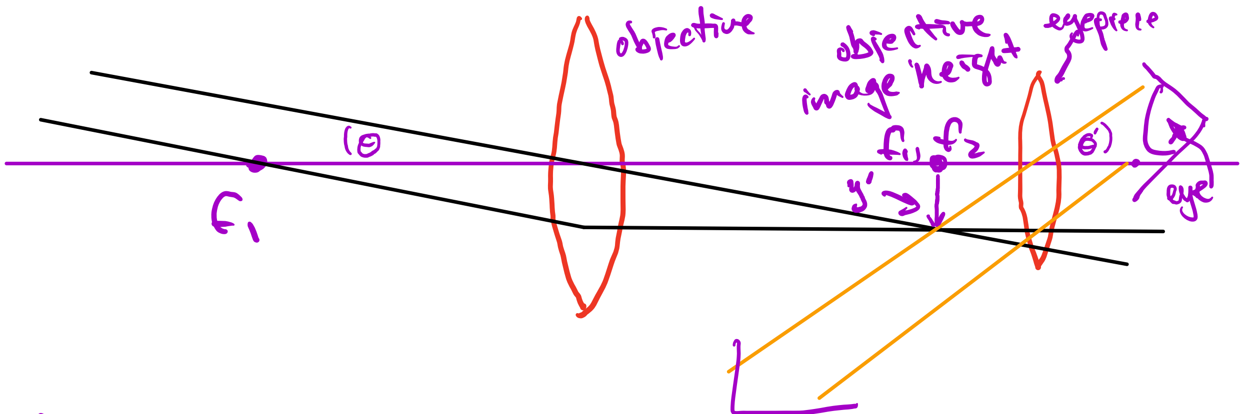


since the eyepiece object is almost at focal pt, light exits eye piece ~ parallel and hits your eye

⇒ eye can relax completely
final image is on retina at an angle that is magnified (follow yellow lines backwards)

This is a telescope!

the eye's lens images the telescope image onto the retina.



final image is at ∞ and is the object for the eye which images onto retina

$\tan\theta = y/s$ y = height of object. for $s \gg y$, $\theta \to$ small

so $\theta = y/s = y'/f_1$

image angle is from eyepiece : $\tan\theta' = y'/s_2$ $s_2 = f =$ object for eyepiece so $\tan\theta' \approx \theta' = y'/f_2$

$M = \dfrac{\theta'}{\theta}$ is the "angular" magnification

$\Rightarrow$ this is the angle subtended by image (not ratio of heights!)

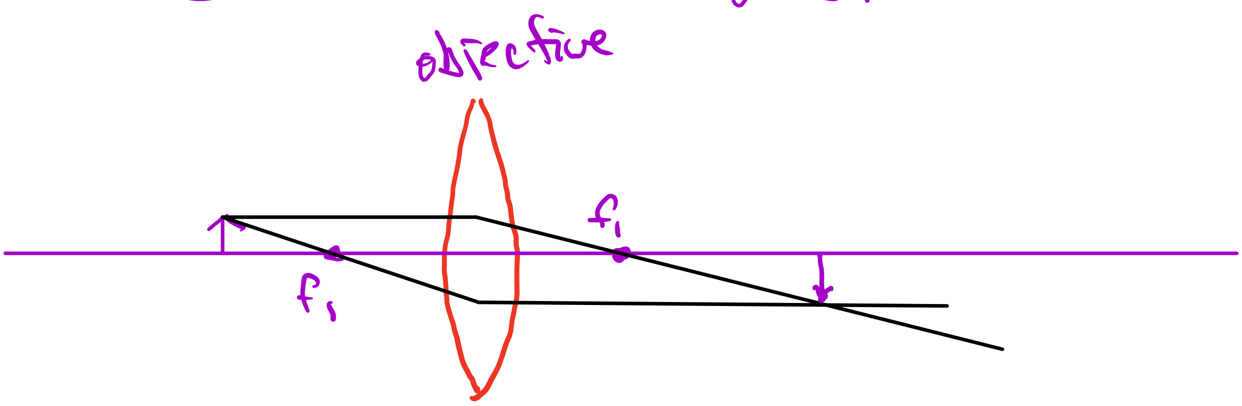
$= \dfrac{y'/f_2}{y'/f_1} = f_1/f_2$

so for telescope want small eyepiece focal pt & large objective focal pt

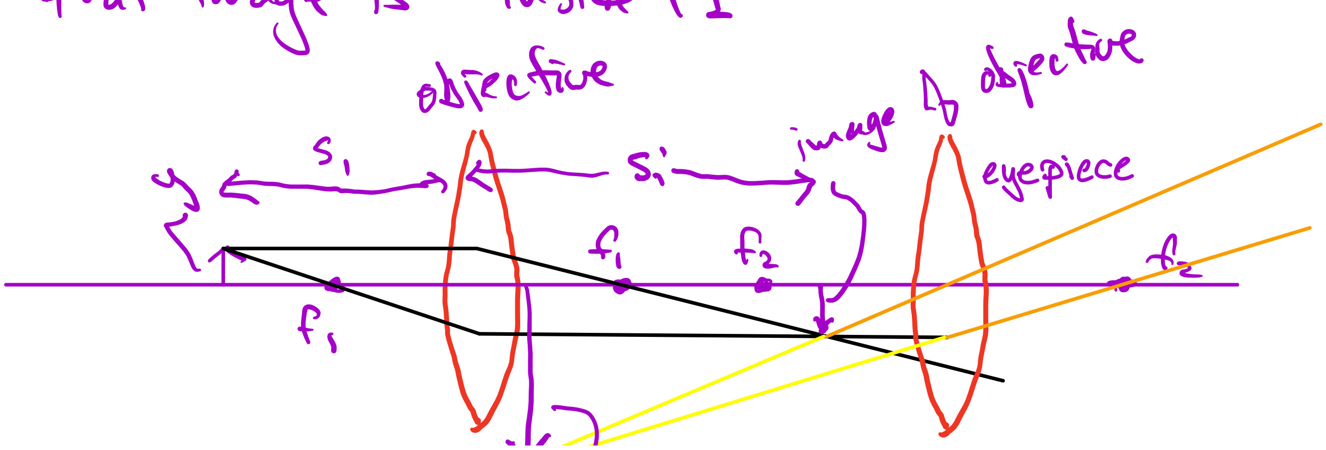
note image is inverted so need mirror to invert

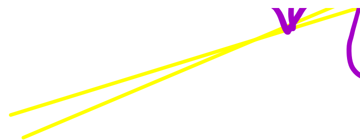
Microscope

here we want the object to be close to the objective, and to also form an image inside the focal pt of eyepiece:



now place converging lens so that image is inside f_2





final image of eyepiece

eyepiece image is inverted & magnified

$$m_{obj} = \frac{h_1'}{h_1} = -\frac{s_1'}{s_1}$$

overall magnification is of objective times magnification of eyepiece, which acts like a magnifier w/ $M_e = \frac{25cm}{f_e}$

$$M = m_{obj} \cdot M_{eye} = \frac{s_1' \cdot 25cm}{s_1 \cdot f_e}$$

usually $s_1 \sim f_o$ so $M = s_1' \cdot 25 / f_o f_e$